Pump wave coherence, modulation instability and their effect on continuous-wave supercontinua

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1. Introduction

An electromagnetic wave that propagates in an optical fibre, where a high optical intensity can be maintained over a long interaction distance, is modified by effects governed by the linear and nonlinear response of the material. The subtle interaction of the nonlinearity and linear dispersion is mediated by the sign of the dispersion. Preserving the quality of the transmitted wave becomes a challenge over long distances, or for increasing intensities above a threshold where the nonlinear response of the medium cannot be neglected. In most situations reducing the onset of strong nonlinearity is necessary, where spectral purity is required; however, in certain cases the converse is true: enhancing the nonlinearity is deliberate in order to exploit the effect of rapid spectral expansion. Supercontinuum light sources, emitting a continuum of frequencies, similar to a thermal source, but with the spatial (and in some cases temporal) coherence, directionality and brightness of a laser has opened lines of scientific enquiry previously closed using conventional methods of illumination.

The availability of high-power fibre lasers, in conjunction with specifically engineered silica fibres with a high nonlinearity and tailorable dispersion profile, has allowed the rapid development of monolithic all-fibre supercontinuum light sources exhibiting extremely broad spectral widths that, in certain cases, address the entire transmission window of silica-glass.

In this paper, I focus on supercontinua generated using a continuous-wave (CW) pump laser resulting in soliton dominated dynamics initiated from modulation instability; specifically analysing the role that intensity fluctuations in the pump source play in the evolution. The outline of the paper is as follows: in Section 2 the basic mechanisms that characterise the evolution of CW pumped supercontinuum are summarised. Section 3 introduces the numerical framework that is used to analyse nonlinear dynamics in single-mode fibres, specifically this section first focusses on the development of a description of the CW pump system, as such initial pump conditions are more complex than the case of pulse pumping. The numerical model is quantitatively compared against a physical CW pump system. Secondly, the well known wave propagation equations are outlined. Recent results exploring the dependence of the continuum formation on the pump coherence are reviewed in Section 4, before conclusions are drawn in Section 5.

The remainder of this introduction provides a brief discussion of general classifications of supercontinuum evolution and motivation for adopting a CW pump format.

1.1. Overview of fibre-based supercontinua

Three broad classes of supercontinuum evolution exist depending on the pump conditions and the fibre parameters [2]: firstly, pumping in the normal dispersion regime, where spectral expansion is governed strongly by self-phase modulation (SPM) for short pulses with sufficient power, and Raman scattering for long pulses (typically >100 ps); secondly, pumping in the anomalous dispersion regime with intense ultra-short pulses, which correspond to low-order solitons that subsequently undergo fission; finally, long-pulse, quasi-CW and CW pumping where large effective soliton-orders correspond to long fission length scales, such that modulation instability (MI) dominates the early dynamics. Such
regimes have been identified, and the dynamics and specific output characteristics of the resulting supercontinuum discussed in detail previously [3,2].

A wave is modulationally unstable subject to anomalous (or negative) group velocity dispersion (GVD) [4], thus in this paper, I consider optical excitation of a nonlinear fibre, where the pump wave experiences only negative GVD. Given this constraint, a reduced subset of evolution dynamics, dominated by soliton effects, can be described in terms of the temporal duration of the input wave.

1.1. Short-pulse pumping

Pumping with pulses that correspond to low-order solitons (approximately $N < 15$ [5,6]) results in dynamics dominated by soliton fission. Practically, this regime is accessed using ultra-short pulses (typically $\leq 100$ fs), with several kilo-Watts peak power. In the fission regime rapid temporal compression (or spectral expansion) in the early stages of the evolution leads to a high degree of temporal coherence; subsequently, perturbations, such as the soliton self-frequency shift (SSFS), continue to degrade the degree of coherence for increasing propagation length [7]. In this regime the time-averaged spectral power is often limited, unless a very high repetition rate pump source is used.

1.1.2. Long-pulse, quasi-CW and CW pumping

For long pulse (i.e. $>1$ ps, or where the effective soliton order $N > 15$ [5,6,2]), quasi-CW or CW pumping dominates the early stages of the continuum evolution. This usually leads to broad supercontinua, with very high average spectral power and spectral flatness, but low temporal coherence.

Although the transition between these regimes is not discrete this classification, based on temporal pump format, provides a useful framework for grouping the complex interactions that characterise supercontinuum generation in fibre. In the case of CW pumping one is firmly in the MI regime. For a fuller discussion of fibre supercontinuum the recent monograph by Taylor and Dudley gives an excellent critique of the subject [8]. In particular the first chapter, Ref. [9], provides an authoritative account of historical developments in supercontinuum generation. In addition, I refer to the review articles by Dudley et al. [3], Genty et al. [5] and Travers [6,2] for a thorough discussion of historical developments up to the state-of-the-art sources, experimental guidelines, and numerical aspects regarding a detailed treatment of the evolution dynamics in various regimes.

In what follows is a brief review of advances in the pursuit of high-average power continuum light sources, pumped with CW sources.

1.2. High-average power continuum light sources

Supercontinuum generated in an optical fibre, pumped by a low (peak) power CW source is typically characterised by high spectral flatness and power [10–22]. Persephonis et al. reported the first significant spectral broadening in a 2.3 km length of Ge-doped fibre, generated by a low-power (Watt level) CW pump source, to which the term supercontinuum could be reasonably applied [23]. However, the rapidly developing interest in pulse-pumped supercontinua, exhibiting in some cases visible spectral content, meant that it was another seven years before the first CW supercontinuum generated in a PCF was reported [10]. Following the work by Avdokhin et al. many new results were reported, using both conventional highly nonlinear fibres (HNLFs) and more exotic PCFs, with tailored dispersion and enhanced nonlinearity [11–20]. This lead to increasing spectral powers and broader bandwidths, including the demonstration of visible supercontinua pumped by a CW laser [17,18,20]. The universal availability of efficient, high-power CW fibre lasers and speciality optical fibres, with highly engineered dispersion profiles, has facilitated all-fibre integrated supercontinuum light sources with spectral powers approaching 100 mW nm$^{-1}$, filling the transmission window of silica [17,20].

Modulation instability, a process inherent to any anomalously dispersive nonlinear medium [24], seeds the formation of high peak power temporal solitons from a low power continuous wave. Sufficiently short duration solitons red-shift through self-Raman interaction, mediated by their local dispersion [25,26]. For certain parameters, MI generated solitons can be too long in duration to undergo significant Raman self-frequency shift, however, energy transfer between temporally and spectrally coincident solitons during collision events can provide the conditions for significant soliton red-shift [25,27]. Such events, have been identified as providing an environment suitable for the generation of localised, large-amplitude, optical rogue-waves, or extreme red-shifted solitons [28,29].

The MI gain in any given fibre (where the magnitude of the GVD and nonlinear coefficient is fixed) is a function of the instantaneous peak power of the incident field. For the purely temporally coherent case of a CW input, the initial peak power is equal to the average power of the field. In practical terms, it is typical for CW fibre lasers to have thousands of longitudinal modes oscillating with no fixed phase relationship [30–32], such modal content gives rise to strong, stochastic temporal intensity fluctuations: the average laser power and instantaneous peak power are no-longer equal. The coherence time provides a quantitative measure of the average duration of the temporal fluctuations in the output of a CW laser, and is inversely proportional to the laser bandwidth. A broader laser spectrum, with a larger range of frequencies exhibits a faster decorrelation time and a shorter temporal coherence [33]. Understanding the relationship between pump wave coherence, modulation instability gain and supercontinuum generation in an optical fibre is the subject of this paper.

2. Overview of the mechanisms involved in CW supercontinuum

Although the dynamics of CW supercontinua have been widely studied and are well understood [22], it is useful to briefly summarise the key points. For an initial CW pump field propagating in the anomalous region of a nonlinear fibre, the evolution can be reduced to three distinct stages [25,2]:

Stage 1. MI induced soliton generation – the decomposition of the initial CW field into a train of ultrashort soliton pulses through modulation instability.

Stage 2. Raman self-scattering of solitons – the short duration, and consequently broad bandwidth, soliton pulses with significant spectral overlap with the Raman gain spectrum experience a red-shift. Energy transfer between solitons due to cross-Raman interaction during collision events can induce enhanced red-shifting of lower frequency solitons. The rate of soliton red-shift is also affected by the local dispersion environment and energy loss from the soliton due to wavelength dependent linear attenuation of the fibre waveguide.

Stage 3. Dispersive wave generation and soliton trapping – dispersive wave radiation in the normal dispersion regime (i.e. at wavelengths shorter than the pump wave) can
be excited if a short duration, MI induced soliton forms sufficiently close to the zero dispersion wavelength (ZDW). Under low-power CW pumping, the duration of the generated MI solitons is not usually sufficient to excite significant dispersive wave components, and typically the supercontinuum, in this case, contains only wavelengths longer than the pump wave. However, strong dispersive wave components can be generated by soliton–dispersive wave interactions: Cherenkov radiation, where the phase-matching condition is met by third-order dispersion and dependent on the nonlinear phase of the soliton; and four-wave mixing (FWM) resonances between the soliton and co-propagating dispersive radiation including the CW pump wave [34]. Interaction of temporally coincident dispersive wave and solitonic radiation through cross-phase modulation (XPM) can cause the dispersive component to move at the same group velocity as the Raman shifting soliton, leading to a blue-shift of the dispersive wave. Significantly blue-shifted continuum from a CW pump can be achieved through the process of soliton trapping, provided group velocity matching is maintained.

Efficient break-down of the pump field through MI requires low anomalous dispersion and a high nonlinearity. The characteristic modulation period, $T_{\text{MI}}$, is defined by the fibre properties, such that [4]

$$T_{\text{MI}} = \frac{2\pi}{\Delta\Omega_{\text{MI}}} = \sqrt{\frac{2\pi^2 |\beta_2|}{P \gamma}},$$

(1)

where $\Delta\Omega_{\text{MI}}$ is the MI frequency, $P$ is the pump power, $\beta_2$ is the GVD and $\gamma$ is the nonlinear coefficient. It can be shown that the characteristic duration, $\tau_0$, of the solitons formed through MI is related to the MI period by [35,22]

$$\tau_0 = \frac{1}{T_{\text{MI}}} \approx 0.1 T_{\text{MI}}.$$

(2)

However, because the MI period is not a fixed quantity when evolving spontaneously from noise, jitter in the MI period leads to MI solitons generated with a distribution of durations. The rate at which the solitons red-shift due to Raman self-scattering (also known as the soliton self-frequency shift) is dependent on their duration:

$$\frac{\partial \Omega}{\partial t} \propto -\frac{|\beta_2|}{\tau_0} \propto -\frac{|\beta_2|}{T_{\text{MI}}} \propto -\frac{\gamma^2 P^2}{|\beta_2|^2}.$$

(3)

The quartic dependence on duration suggests a rapid increase in the rate of SSFS for shorter duration solitons. The distribution in the duration of the solitons generated from MI leads to a distribution in the rate of SSFS. Thus, the jitter in the MI period leads to a degradation in the phase and intensity stability of the generated continuum, and high shot-to-shot spectral variation. Integration of the power spectrum, using an optical spectrum analyser, produces a characteristically smooth response over the bandwidth of the continuum, as the spectral variation is averaged over a large number of shots. The phase stability of long-pulse and CW pumped supercontinua can be enhanced by inducing the MI [36–38].

Consideration of the stages outlined above is useful in the design of a CW supercontinuum source: for efficient MI, short duration solitons and consequently maximum SSFS requires a low $|\beta_2|/\gamma$ ratio. Moving close to the ZDW, lowering the magnitude of the GVD at the pump wavelength, satisfies this requirement. This also benefits stage three as short duration solitons, with broad bandwidth, propagating close to the ZDW can excite dispersive wave radiation. However, moving closer to the ZDW can often prevent any significant red-shifting of solitons due to a positive dispersion slope, although the effect of Raman gain has been shown to extend the SSFS in certain cases [39]. Thus, the conditions for efficient red and blue-shifted continuum in this pump regime are conflicted.

In addition to the fibre parameters, the efficiency of MI, in media with instantaneous nonlinearity, is known to be affected by the degree of wave coherence. One model for the frequency dependent MI gain in optical fibre, in the presence of partial coherence can be written [40]

$$g(\omega) = |\beta_2|\omega \left( \frac{B_2 P}{|\beta_2|^2} - \omega^2 \right)^{1/2} - 2 |\beta_2| \omega^2 \sigma,$$

(4)

where $\sigma$ is the spectral bandwidth of the pump source. Eq. (4) suggests that the MI gain decreases for increasing partial wave coherence, i.e. as the bandwidth of the pump wave increases; this dependence is shown in Fig. 1, where the maximum gain occurs for the most coherent pump wave. It is perhaps surprising that even in the limit of infinitesimal pump bandwidth (i.e. $\sigma \rightarrow 0$), the gain is still greater than the coherent case.

Given that for CW pump cases, MI is a precursor to soliton formation and supercontinuum, it would be reasonable to postulate that for a given fibre the broadest continuum spectrum would form under conditions where MI gain was maximised, with the above analysis predicting maximum MI growth for the most coherent pump wave. However, it has been observed experimentally that moderate partial wave coherence initially enhances continuum formation [41]. For increasing partial coherence continuum formation is inhibited, as is expected from Eq. (4). The study by Martinez-Lopez and co-workers used three different CW pump systems, with equal centre wavelength and output power, but different spectral bandwidths (~0.02 nm, ~0.22 nm and ~1.0 nm, respectively), to excite a nonlinear response in a length of dispersion-shifted fibre (DSF). This work showed that the broadest continuum was formed using the source with a bandwidth of ~0.22 nm.

This empirical result is perhaps surprising if one considers Eq. (4), that says that the maximum MI gain occurs for the most coherent pump source, given the central role MI plays in the initial formation of one-solitons from a CW input wave [42]. One explanation for the discrepancy is that an initial enhancement to the instantaneous peak power, due to moderate partial coherence, is not fully accounted for in this model of incoherent MI. In addition, this clearly suggests that an optimal value of pump coherence exists that maximises the effect of increasing the instantaneous peak power through intensity fluctuations, but remains sufficiently coherent, such that the MI is not suppressed according to Eq. (4).

![Fig. 1](image)

Fig. 1. The frequency dependent MI gain in the presence of partial coherence, given by Eq. (4). The parameters are: $\beta_2 = -0.82 \text{ ps}^2 \text{ km}^{-1}$, $\gamma = 20 \text{ W}^{-1} \text{ km}^{-1}$, $P = 3.75 \text{ W}$ and $\sigma$ is as indicated. The black curve shows the frequency dependent MI gain in the coherent case.
3. Modelling

Engineering of nonlinear effects in fibre involves optimisation over a complex space of parameters, thus numerics involving computation modelling are widely employed to direct physical experimentation. Numerical simulations provide an ideal laboratory environment equipped with perfect experimental diagnostics, and are an invaluable tool provided they are empirically validated.

3.1. A CW pump source

Before considering the evolution of the CW pump field in a HNLF, under the influence of linear and nonlinear effects, it is necessary to consider the exact nature of the initial conditions, and ensure that they well match the parameters of a physical pump system.

Accurately describing the phase and amplitude fluctuations of a CW fibre laser, containing a very large number of coupled modes, is a complex problem and a number of models have previously been developed [43–45,31,46,32]. Reducing the problem to a strictly forward propagating model, eliminating the need to consider cavity mode effects, allows the use of standard unidirectional nonlinear Schrödinger-type solvers to simulate the nonlinear evolution of the complex field. Physically this corresponds to a correctly isolated amplified spontaneous emission (ASE) based CW pump scheme.

The components of a typical high-power ASE-based CW system, used as the pump source in supercontinuum experiments is shown in Fig. 2. In this case, the source is based on a chain of amplifiers intersected by bandpass filters. ASE generated in the first-stage amplifier is filtered, amplified and filtered again before passing a final stage power amplifier. Such a system can be realised using various doped-fibre amplifiers. Typically Yb-doped or Er-doped amplifiers, both mature technologies and widely available as packaged commercial units, are used for operation in either the 1.06 µm or 1.55 µm band. Recent experimental investigation has shown that the properties of supercontinua generated using both CW ASE-based and laser-type pump sources of equivalent bandwidths. It is clear from these figures that as the bandwidth of the pump source increases, the rate of fluctuations in the time-domain field also increases. The temporal coherence is a quantitative measure of this rate of fluctuation. The temporal coherence, $\tau_c$, of a light source, with a complex electric field $E(t)$ can be determined by considering the field autocorrelation (AC) given by

$$\tau_c \approx |\langle I(t)I(t-\tau)\rangle|$$

where $I(t)$ is the second-order coherence function [48], which is the Fourier transform of the spectral power $S(\omega)$ [33]. This relation clearly links the pump spectral bandwidth inversely with the coherence time. However, it says nothing about the temporal intensity fluctuations: all of the spectral width could be (momentarily) due to phase fluctuations.

The intensity AC, however, of a complex, or highly structured waveform, such as a CW laser, given by [48]

$$A^2(\tau) \approx |F^2(\tau)|^2 + \int_{-\infty}^{\infty} I_{\text{env}}(t)|I_{\text{env}}(t-\tau)|dt$$

where $I_{\text{env}}(t)$ is the intensity of the waveform envelope, contains information about the intensity fluctuations. For a purely CW signal, the intensity AC will be a flat line, but for a more general signal, the

![Fig. 2. Typical components of a tunable ASE-based CW pump source, with temporal coherence control through bandwidth variation. DF, Er-doped fibre amplifier; BPF, bandpass filter ($\Delta \lambda = 12$ nm); TBPF, tunable bandpass filter (0.1 < $\Delta \lambda$ < 15 nm).](image-url)
AC function possesses a flat pedestal half the height of the peak, and a spike, symmetric about the zero delay \((\tau = 0)\) characterising the average-duration of the finest structure of the signal noise. Although it is not possible to measure the full field of a CW source, with currently available techniques, (as it is for fs-scale pulses using frequency resolved optical gating (FROG) techniques, among others\([49,50]\)) quantitative agreement between modelling and experiment can be confirmed using a combination of intensity autocorrelation to establish the average duration of the temporal intensity fluctuations, and acquisition of the optical spectrum. Fig. 5 shows the corresponding measured and calculated intensity autocorrelation functions for a range of bandwidths (as indicated) of the physical system, realised using the configuration outlined in Fig. 2; calculations were based on the temporal fields generated using the model outline above. Excellent agreement confirms the approach used to create the initial CW fields, including an empirically valid description of the fluctuations in the temporal intensity.

The model can be used to quantify the enhancement to the instantaneous peak power through increased modal interaction as the bandwidth of the pump source broadens and the temporal coherence degrades. It is clear from Fig. 3 by inspecting the ratio of the peak to average power, that the rate of increase slows. The exact logarithmic relationship between peak power enhancement and pump source bandwidth is plotted in Fig. 6.

The logarithmic fit shows a strong saturation of the peak power enhancement for broadening pump bandwidth. This suggests that beyond a critical pump bandwidth the efficiency of continuum generation will reduce significantly as the effective peak power enhancement is lost, and the MI gain is degraded by incoherence. Plotted also in Fig. 6 is the dependence of the temporal coherence on pump bandwidth, showing a rapid decay for increasing partial coherence.

### 3.2. CW continuum evolution in a highly nonlinear fibre

Modelling of supercontinuum evolution in optical fibre waveguides can be well described by the one-dimensional generalised

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**Fig. 3.** Numerically modelled temporal intensity profiles of the ASE-based CW pump source for three pump bandwidths as indicated. The time averaged power is shown with a dotted blue line, and is constant with increasing pump bandwidth to within 1% of a target value of 6.3 W. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

**Fig. 4.** Numerically modelled spectral intensity profiles of the ASE-based CW pump source for three pump bandwidths as indicated. Note a change of x-axis scale in figure (c).

![Graph A](image1.png)

- **(a) Pump bandwidth: 0.13 nm.**

![Graph B](image2.png)

- **(b) Pump bandwidth: 1.34 nm.**

![Graph C](image3.png)

- **(c) Pump bandwidth: 4.02 nm.**
The nonlinear Schrödinger equation (GNLSE) that includes the relevant linear and nonlinear contributions that modify the initial field depending on the parameters of the fibre, such as GVD and nonlinearity [4]. A frequency domain formulation of the equation can be written as [51,52]

\[
\frac{\partial \tilde{A}(z, \omega)}{\partial z} + \left[ \frac{\chi(\omega)}{2} \sum_{\omega_k} \frac{\beta_k}{k!} (\omega - \omega_k)^k \right] \tilde{A}(z, \omega) = i \gamma \frac{\omega}{\omega_0} \mathcal{F} \left[ A(z, T) \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 dT' \right]
\]

Fig. 5. Second harmonic intensity autocorrelation traces (measured from experiment and calculated from simulation) of the CW input pump source for three pump bandwidths: 0.36 nm (a and b); 1.77 nm (c and d); 5.28 nm (e and f).

Fig. 6. The relative peak power enhancement factor (where \( \Psi_{\text{enhancement}} = P_{\text{peak}} / P_{\text{average}} \)) as a function of the FWHM pump source bandwidth.

\[1\] It is often preferred to derive a time-domain GNLSE equation, because of analytic similarity to the NLS equation – about which there is vast literature [53]; but in fact a frequency domain formulation has many advantages: allowing more direct treatment of frequency-dependent effects such as dispersion, dispersion of the nonlinearity, loss and the fibre effective mode area [51,52].
where $\tilde{A}(z, \omega)$ is the (linearly polarised) complex spectral amplitude of the pulse envelope, which is a function of the propagation distance $z$, within a retarded time frame $T = t - z/v_g$, moving at the group velocity $v_g$ of the pulse. The nonlinear coefficient is defined in the usual way: $\gamma = \omega n_2 A_{\text{eff}}/(C_0c)$, with units of $W^{-1} \text{km}^{-1}$, where $n_2$ is the nonlinear refractive index, $c$ the speed of light and $A_{\text{eff}}$ the effective mode area.

In Eq. (9), the left hand side terms model linear effects: the power attenuation $a$; and the dispersive coefficients $b_j$ to arbitrary order – however, in results discussed in Sections 4.2 and 4.3 only GVD ($b_2$) will be included; the role of third-order (TOD) dispersion ($b_3$) on continuum formation will be briefly discussed in Section 4.4. The right hand side describes nonlinear effects: the instantaneous contributions to the nonlinearity caused by the electronic Kerr effect, self-steepening and optical shock formation; and the delayed contribution from non-instantaneous effects, namely inelastic Raman scattering (Brillouin scattering is not included due to the assumption of unidirectional propagation; in most experimental cases stimulated Brillouin scattering (SBS) is not excited due to incoherence of the pump wave). The convolution integral contains an instantaneous electronic and a delayed Raman contribution, given by

$$R(t) = (1 - f_R) \delta(t) + f_R h_R(t)$$

where $\delta(t)$ is the Dirac delta function, $f_R = 0.18$ is the fractional contribution of the delayed Raman response, and the response function $h_R(t)$ can take a number of forms, depending on the complexity (and accuracy) of the desired function $[54, 4, 55]$; a multi-

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Fig. 7. Optical spectra and corresponding measured background-free (non-colinear) intensity autocorrelation trace for three increasing pump source bandwidths: 0.36 nm (a and b); 1.77 nm (c and d); 5.28 nm (e and f).
vibrational-mode model, developed by Hollenbeck and Cantrell [55], was used throughout.

It has already been stated that models of CW systems are going to be approximations, capturing a snap-shot of the dynamics on a finite temporal grid. The problem of supercontinuum evolution is made computationally more demanding due to the requirement to satisfy the Nyquist sampling theorem and obtain a suitable frequency coverage to contain the spectral expansion of the input field on propagation. A temporal grid, discretised with $2^{17}$ points, was used resulting in long computation times for even modest propagation distances. In addition, because CW continuum evolution is seeded by noise driven processes, it is necessary to perform ensemble simulations, with different random initial pump conditions, and subsequently average over the ensemble for good quantitative agreement between numerical and physical experiment [7]. Typically >100 realisations constituted an ensemble.

The recent monograph on supercontinuum by Taylor and Dudley [8,52], and references therein, provides a more detailed discussion of formulations of, and solutions to the GNLSE in the context of modelling supercontinuum generation in optical fibres.

4. The dependence of pump coherence in the development of a CW continuum

An overview of recent experimental and numerical results [1] are discussed in this section.

4.1. Experimental details

Using a broadband cascaded ASE-based, high-power CW source, incorporating a widely tunable filter (with a tunable passband bandwidth of $0.1 < \Delta \lambda \leq 15$ nm), it is possible to control the pump source coherence in a range corresponding to a coherence time of $\sim 20$ ps $\geq \tau_c \geq 50$ fs. Such a system, as outlined in Fig. 2, was realised using two low-power Er-doped fibre amplifiers (with gain bandwidths covering 1.545–1.575 $\mu$m) and a 10 W power amplifier optimised for operation at 1.565 $\mu$m.

The lower limit on the available temporal coherence (or the broadest available pump bandwidth) was restricted by the parabolic gain shaping of successive stages of amplification. The strong gain shaping can be seen in Fig. 7 where the optical spectrum, and corresponding intensity autocorrelation function, is plotted for three increasing bandwidths. Greater than 30 dB suppression between the signal and pedestal was achieved across the range of coherence times accessible using this source.

The predicted maximum bandwidth, calculated using the ASE-based model of the pump system, is shown in Fig. 8. The black vertical dotted line (at 30 nm) depicts the 3 dB gain bandwidth of the perfect parabolic profile assumed for the three Er-doped fibre amplifiers in the chain, and the blue dotted line (at 15 nm) denotes the maximum passband of the physical tunable filter.

It is clear that as the passband of the filter is increased arbitrarily, the pump bandwidth no-longer broaden due to the finite gain bandwidths and successive stages of amplification. In addition, the resulting output pump bandwidth is much less than the 3 dB gain bandwidth of a single amplifier. The maximum accessible bandwidth is estimated to be $\sim 11$ nm. In fact, the physical system was limited to $\sim 7$ nm due to non-perfect overlap of the gain profiles in the amplifier stages.

The ASE-based CW pump source was coupled to a length of HNLF through a high-power inline optical isolator to prevent back reflected light damaging upstream components of the pump system, due to the high-gain of the final stage amplifier (see Fig. 9).

The ASE-based model of the pump system, is shown in Fig. 8. The black vertical dotted line (at 30 nm) depicts the 3 dB gain bandwidth of the physical tunable filter.

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The spectral output of the HNLF, under a fixed pump power of 6.3 W, for pump bandwidths in the range 0.3–7 nm, was recorded using an optical spectrum analyser. The experimentally measured, and numerically calculated continuum widths (10 dB) as a function of the pump source bandwidth (3 dB) are plotted in Fig. 12.

The first observation is that there is strong agreement between the measured and calculated data. The second observation is that the trend agrees with the analysis and discussion above: the
optimum condition for efficient continuum expansion under CW pumping is not the nearly coherent case, as suggested by Eq. (4). For increasing partial coherence, the generated continuum width increases with increasing pump bandwidth. Beyond a pump bandwidth of $\sim 3$ nm the rate of increase in the width of the corresponding output continuum slows and the spectral expansion begins to saturate. Indeed beyond $\sim 5$ nm the spectral expansion appears to start to contract. The effect of saturation and the transition to contraction in the continuum width is more convincing in Fig. 12b, where broader bandwidths were accessible with the idealised numerical model. However, the peak continuum width value is less well defined in the numerical data because of limited ensemble size averaging.

Insight can be gained from inspection of the input temporal field intensity and the resulting output continuum in the spectral domain, calculated using the numerical model (see Figs. 13 and 14). The MI period for the given fibre, wavelength and power is represented in Fig. 13 with a horizontal bar, for reference. When the average duration of the intensity fluctuations in the input field far exceed the MI period (Fig. 13a), i.e. when the pump field is nearly coherent, the instantaneous peak power enhancement is low and the MI gain is limited, leading to no continuum formation (Fig. 14a). Conversely, if the average duration of the fluctuations is much less than the MI period (Fig. 13c) the pump field is too incoherent for efficient MI, and continuum formation is effectively suppressed (Fig. 14c). However, when the period of the fluctuations is equivalent to the MI period (Fig. 13b), partial coherence results in an enhancement to the instantaneous peak power, while the pump remains sufficiently coherent for MI gain to not be fully inhibited. In fact, as long as the pump coherence time is longer than the MI period intensity noise fluctuations enhance the soliton energies formed through MI. In this case a broad continuum can be formed (Fig. 14b). In addition, Fig. 14b shows the largest energy transfer to dispersive waves around 1.3 $\mu$m, suggesting that the most intense and short duration solitons were formed in this case [56,57].

Fig. 15b shows the calculated spectral evolution for three input pump bandwidths, as indicated, where the initial noise field is generated using the ASE source model. However, in order to illustrate a full contraction in the continuum dynamics as the bandwidth of the pump field increases beyond that which remains sufficiently coherent for MI to occur, the condition on the model to exactly match the parameters of the physical pump system was relaxed.

Further insight can be gained from calculation of the field spectrum [48,6]. The output spectrograms (after the full 50 m propagation) of the corresponding evolution plotted in Fig. 15 are shown in Fig. 16. Solitonic features are represented as intense localisations in time–frequency space, and can be seen to have red-shifted under the action of self-Raman interaction. Dispersive features are characterised by lower intensity and often a temporal chirp. It is not surprising that the pump bandwidth that generated the broadest output spectrum (see Fig. 15b) shows the greatest soliton content in the spectrogram representation (see Fig. 16b). Each intense soliton can be mapped to a dispersive wave resonance around 1.3 $\mu$m generated by mixing with the pump line, indicating that short duration, broad bandwidth solitons have been generated from MI in this case. The two panels of Fig. 16 corresponding to the extreme pump bandwidths show little soliton content, and consequently limited spectral expansion after the full propagation distance.

### 4.3. Extending the space of parameters

It has been shown empirically that, contrary to recent theory of incoherent MI, the optimum condition for the generation of short duration solitons from MI is not the nearly coherent case. In fact, moderate partial coherence has been found to have a dramatic effect on the efficiency of spectral expansion of a CW pump field in a HNLF [41,1]. While this is an important realisation, it is necessary to understand where the optimum exists: at what point does the pump field become too incoherent, and fully suppress MI gain?

#### 4.3.1. A simplified model of the CW pump source

Although the CW ASE-based model of the pump source outlined in Section 3.1 shows excellent agreement when compared to the physical pump system, i.e. provides empirically valid initial pump...
noise conditions suitable for use in the nonlinear propagation equation (given by Eq. (9)), it is computationally intensive and prevents the broad sweeping of a parameter space necessary to uncover a quantitative relationship between pump source bandwidth and efficient continuum generation.

Fig. 12. The dependence of the generated continuum width (10 dB) on the CW pump bandwidth (3 dB). The average pump power was 6.4 W and the HNLF length was 50 m.

Fig. 13. Temporal input field intensities for three pump bandwidths, computed using the CW model: 0.33 nm (a); 2.58 nm (b); 6.24 nm (c). The horizontal bars show the modulation instability period, $T_{MI}$, for the HNLF fibre parameters, pump wavelength (1.55 μm) and power (6.3 W) corresponding to the experimental conditions.

Fig. 14. The corresponding computed single-shot spectra after propagation in the 50 m length of HNLF: pump bandwidths 0.33 nm (a); 2.58 nm (b); 6.24 nm (c). The spectral input pump lines are shown with a dotted curve.
A simplified model of a CW-fibre laser, not involving numerical evaluation, is given simply by a sech-shaped spectral intensity profile and an associated random spectral phase [22], using an approach similar to Ref. [43]. Although this model exhibits an unphysical dependence on the numerical grid size and ignores any phase relationship between laser modes (accounted for in the previous description of the CW pump source), it is self-consistent for simulations conducted over numerical grids of constant size.

Fig. 17 shows the calculated intensity autocorrelation of the input field generated using the analytic model described above, for three pump bandwidths (as indicated). Shown for comparison are the corresponding autocorrelation functions computed using the numerical model described in Section 3.1 (and plotted previously alongside the measured data in Fig. 5).

It is evident that superior quantitative agreement is achieved using full numerical simulation. However, the simple analytic expression provides a convenient and rapid means of acquiring a large number of suitable initial noise fields for large-scale ensemble simulations over a broad parameter space. In all proceeding numerical results presented the analytic model of a CW laser is used.

4.3.2. Finding the optimum bandwidth

Fig. 18 illustrates that a clear optimum pump bandwidth generates the broadest spanning continuum in a fixed fibre with constant average pump power. The results are illustrated on a wavelength scale centred at 1.065 μm, with 10 W average power in a fibre with the following parameters: $\beta_2 = -0.012 \, \text{ps}^2 \, \text{km}^{-1}$; $\gamma = 44 \, \text{W}^{-1} \, \text{km}^{-1}$; and length $L = 20 \, \text{m}$. These reflect typical
parameters for CW continuum experiments conducted using a Yb fibre laser and a photonic crystal fibre (PCF), but the results are generally applicable. Noise is included using the standard model equivalent to one photon per mode (as previously) [3]. Eq. (9) is used to propagate the complex field in the HNLF; high-order dispersion terms are not included in this case.

It can be seen in Fig. 18 that for the given fibre parameters, a pump bandwidth of approximately 2 nm generates the broadest spanning continuum. This corresponds to a pump coherence time of ~1.7 ps, compared to an MI period of 0.74 ps. In the construction of a CW supercontinuum light source, control of the dispersion and nonlinear parameters of the HNLF is important to stimulate and

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**Fig. 16.** Single-shot spectrograms after the full 50 m propagation length of the HNLF for three input pump bandwidths: 0.56 nm (a); 4.25 nm (b); 38.66 nm (c). Colour scale: -90 dB 0 dB. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
optimise the evolution in accordance with the stages outlined in Section 2. In addition, according to Fig. 18, the linewidth of the pump source should be that which is approximately equivalent to three times the MI period for optimum spectral expansion and efficient continuum formation. To determine the generality of this constraint and establish this relationship as a universal design guideline the dependence of the continuum efficiency on the pump bandwidth was studied in a range of fibres, with a distribution of MI periods. For a fixed nonlinear coefficient and pump power the nonlinear length, \( L_{\text{NL}} \), is a constant quantity. Variation of the GVD parameter results in a linear progression in \( O_{\text{MI}} = \sqrt{c/b^2} \), which is proportional to the MI period.

Fig. 19 shows the profile of the continuum width at the 20 dB level, generated in a range of fibres, as a function of the pump bandwidth. Each curve consists of 20 pump bandwidths logarithmically distributed between 0.03–30 nm, with a distribution of MI periods. For a fixed nonlinear coefficient and pump power the nonlinear length, \( L_{\text{NL}} \), is a constant quantity. Variation of the GVD parameter results in a linear progression in \( O_{\text{MI}} = \sqrt{c/b^2} \), which is proportional to the MI period.

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The first observation is that the continuum width scales with the quantity \( O_{\text{MI}} \). This is consistent with the overview given in Section 2, where for efficient continuum formation a low value of GVD and high nonlinearity is desirable. This can be explained by a decrease in the induced soliton duration as the MI period reduces, with a positive scaling in \( O_{\text{MI}} \). As the MI period is reduced the tolerated pump incoherence is higher and hence the range of pump bandwidths over which a continuum will form should increase with increasing \( O_{\text{MI}} \); this is clear from Fig. 19: the curves become wider with increasing \( O_{\text{MI}} \). Perhaps the most striking observation is that the peak value shifts to broader pump bandwidths with an increasing value of \( O_{\text{MI}} \).

Thus, the optimum bandwidth can be expressed as a function of the MI bandwidth. The relationship, based on data extracted from Fig. 19, is plotted in Fig. 20, and fitted with a simple linear function. The optimum bandwidth is approximately one third of the MI bandwidth: \( \Delta \omega_{pump, \text{optimum}} \approx \frac{1}{3} \Delta \omega_{\text{MI}} \). If the coherence time of the sech-shape pump spectrum is given by \( t_c \approx 3T_{\text{MI}} \), the optimum pump coherence time can be expressed as \( t_c \approx 3T_{\text{MI}} \).
4.4. The role of third-order dispersion

All the analysis thus far has neglected higher-order dispersion terms, however, the generated frequencies of a continuum, particularly in the case of optimum pump coherence, would be outside the region of flat dispersion in any real HNLF fibre, such as PCF. Here, the role of third-order dispersion is briefly discussed.

In the case of HNLFs, such as PCFs with a single zero dispersion wavelength (ZDW), positive third-order dispersion, typically encountered in practical situations, can restrict continuum formation [22,58]. This is due to the soliton condition: if loss is negligible and nonlinearity is constant, the soliton duration has an inverse relationship with dispersion. Consequently, as solitons red-shift, under the influence of the SSFS, they temporally broaden and reduce in bandwidth, affecting the rate at which they Raman shift. Contrastingly, for PCFs with a double ZDW, when the pump wavelength lies relatively far from the first ZDW, the ratio of $\beta_2/\gamma$ is approximately constant with wavelength. This means that the solitons emitted through MI remain short in duration enabling efficient SSFS [16].

The effect of restricted red-shift, as a result of positive TOD, can be seen in Fig. 21, where the peak value is reduced by approximately a factor of two and a half. The second observation is that the width of the efficiency profile has changed as a result of including the $\beta_3$ term: initially as the coherence of the pump source degrades the rate of increase in the width of the generated continuum is slower than in the case when $\beta_3 = 0$; similarly the peak value is less well defined. This suggests that for large positive values of TOD (i.e. if the pump wave is close to the ZDW of the HNLF), the influence of the pump coherence on the modest continuum formed is lower because the enhancements to the soliton energies due to pump intensity noise are damped by the effect of positive TOD as the soliton tries to rapidly red-shift.

5. Conclusion

In conclusion, the role of pump wave coherence and its effect on MI and the generation of CW continuum has been reviewed in detail. Guidelines for the efficient generation of broadband light from a partially coherent CW source have been outlined. It has been shown specifically that for the case of supercontinuum evolution from an initially CW wave, where MI is the dominant mechanism leading to soliton effects, intensity fluctuations in the pump wave are shown to initially enhance and ultimately inhibit continuum formation. The optimum degree of pump coherence is that which supports the largest peak power enhancement, due to strong modal interactions, that remains sufficiently coherent, such that MI is not fully suppressed. Practically, this corresponds to a pump coherence time approximately one third of the MI period, defined by the specific HNLF parameters used for generation. This conclusion is contrary to current theory of incoherent MI, which suggests that the optimum condition for maximum MI gain is the nearly coherent case. For a HNLF with strong TOD at the pump wavelength, it was shown that the rate of reduction in continuum efficiency, away from the optimum value of pump coherence, decreased.

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